

Microscopic Properties of Horizons

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Abstract

We suggest that all horizons of spacetime, no matter whether they are black hole, Rindler, or de Sitter horizons, have certain microscopic properties in common. We propose that these properties may be used as the starting points, or postulates, of a microscopic theory of gravity.

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It is a curious historical fact that progress in physics is often made when a fundamental problem is raised to the status of a postulate. Something like that was done by Jacobson in 1995.[1] In that time theoretical physicists were very much puzzled by the result that the entropy of a black hole is, in natural units, one quarter of its horizon area, and several explanations, based either on string theory [2] or canonical quantum gravity [3], were provided some time later. Instead of attempting to provide yet another explanation Jacobson assumed that not only black hole horizon, but also the so called Rindler horizon of an accelerating observer may be associated with an entropy which is one quarter of its area. Using this assumption, together with the first law of thermodynamics, Jacobson was able to *derive* Einstein's field equation describing the interaction between spacetime and the Unruh radiation observed by an accelerated observer.[4]

In more precise terms, Jacobson's line of reasoning, with slight modifications of the original idea, may be expressed as follows: Unruh radiation coming, from the observer's point of view, through the Rindler horizon, carries energy and momentum which may be stored to the observer's detector, and the detector becomes heated. As a result, spacetime in the vicinity of the observer becomes curved and, consequently, the paths of the light rays determining the Rindler horizon change. A closer investigation reveals that, in the rest frame of the observer, the area of the part of the horizon considered by the observer shrinks during the radiation process. The Unruh radiation with temperature T obeys the first law of thermodynamics:

$$\delta Q = T\delta S, \tag{1}$$

where δQ and δS , respectively, are the changes of the heat and the entropy of the detector due to radiation. If one assumes that between the change δA of the horizon area, and the entropy change δS there is the relationship

$$\delta S = -\frac{1}{4}\delta A, \tag{2}$$

then Eq.(1) gives the relationship between the energy momentum stress tensor $T_{\mu\nu}$ of the radiation (δQ depends on $T_{\mu\nu}$), and the Ricci tensor $R_{\mu\nu}$ of spacetime (δA depends, through Raychaudhuri equation, on $R_{\mu\nu}$). Actually, Eq.(1) gives

precisely Einstein's field equation, which thereby has been derived by means of purely thermodynamical arguments.

Jacobson's thermodynamical derivation of Einstein's field equation, based entirely on the first law of thermodynamics, and the assumed relationship (2) between entropy and horizon area, raises an interesting question of a possibility that maybe Einstein's general theory of relativity describes just thermodynamics of spacetime and matter fields. In that case the spacetime metric $g_{\mu\nu}$ is probably not a fundamental field of nature, and all attempts to quantize Einstein's equation canonically would be, to quote Jacobson's words, "no more appropriate than it would be to quantize the wave equation for sound in air".[1]

The thermodynamical properties of any system follow from the statistical mechanics of that system which, in turn, follows from its microscopic properties. It would be very interesting to find the physical laws governing the microscopic properties, and hence the statistical mechanics, of spacetime, but if Jacobson's provocative statement is true, a straightforward application of the rules of quantum mechanics to Einstein's field equation (canonical quantization, for instance) is of no help. At the present state of research we must just postulate those laws. The laws must be postulated in such a way that, among other things, Einstein's field equation is produced in the thermodynamical limit. For the sake of simplicity we shall assume that spacetime, at least effectively, is described by a four-dimensional (pseudo) Riemannian manifold. An advantage of this assumption is that the whole tensor machinery of Riemannian geometry is still in our service.

The crucial step in Jacobson's derivation of Einstein's field equation was the relationship (2) between horizon area and entropy, and our task is to find the simplest possible postulates which imply that relationship. During the past thirty years or so Bekenstein and others have produced an enormous amount of evidence supporting the proposal that the area of the event horizon of a black hole has an equal spacing in its spectrum.[5] Our first postulate therefore reads:

Postulate 1: *When one measures the area of a horizon, or any part of it, the possible outcomes of measurements are:*

$$A_n = n \cdot A_0, \quad (3)$$

where n is a non-negative integer, and A_0 is a constant.

Now, the observer may measure the whole area of the horizon by dividing the horizon into parts, measuring the area of each individual part, and finally adding together the results of measurements. For the horizon area $A = nA_0$ the maximum number of parts is n . It is a nice exercise of combinatorics to show that the number of different ordered p -tuples

$$(m_1, m_2, \dots, m_p)$$

such that $1 \leq p \leq n$, $m_j \in \mathcal{Z}^+ \forall j = 1, 2, 3, \dots, p$, and

$$m_1 + m_2 + m_3 + \dots + m_p = n, \quad (4)$$

is

$$\Omega(n) = 2^{n-1}. \quad (5)$$

In other words, the areas of the individual parts of the horizon may be summed over to nA_0 in 2^{n-1} ways. Each ordered p -tuple represents a certain combination of the areas of the parts of the horizon with fixed total area. We identify each such combination as a microstate of the horizon. Hence we get the following postulate:

Postulate 2: *The number of microstates corresponding to the same macrostate of the horizon is equal to the number of different combinations of the areas of its parts.*

Our Postulates 1 and 2 imply that the horizon has entropy S_h which, for macroscopic horizons, is proportional to the area:

$$S_h = k_B \ln \Omega(n) = (n-1)k_B \ln 2 \approx \frac{k_B \ln 2}{A_0} A_n, \quad (6)$$

where k_B is Boltzmann's constant. For horizons having infinite area, such as the Rindler horizon, this entropy may be associated with the considered finite part of the horizon. In the process we have introduced the area A_0 which may be viewed, in our approach, as a fundamental constant of nature. The requirement that the entropy of the horizon is, in natural units, one quarter of its area, gives the following relationship between Newton's gravitational constant G and the area A_0 :

$$G = \frac{A_0 c^3}{4\hbar \ln 2}, \quad (7)$$

and therefore

$$A_0 \approx 7.23 \times 10^{-70} m^2. \quad (8)$$

Although our Postulates 1 and 2 imply that the entropy of the horizon is proportional to its area, they say nothing about the entropy of the radiation emitted by the horizon. Therefore we state:

Postulate 3: *In thermal equilibrium the sum of the entropies of the horizon and the radiation is constant.*

In other words, the entropy of the horizon decreases exactly as much as the entropy of the radiation increases. As a whole, our Postulates 1, 2 and 3 imply the relationship (2) between the horizon area, and the entropy of radiation.

There is only one part missing from our set of postulates. To be able to derive Einstein's field equation from the first law of thermodynamics of Eq.(1) we need a postulate which tells that the radiation emitted by the horizon has a certain temperature. Since entropy is proportional to area, and between energy and entropy there is the relationship given by Eq.(1), we need a postulate which tells the relationship between the area of the horizon, and the amount of energy which may be extracted out from the horizon.

As it is well known, the temperature of the radiation emitted by any horizon is, in SI units, [4]

$$T = \frac{\hbar \kappa}{2\pi k_B c}, \quad (9)$$

where κ is the surface gravity of the horizon. Hence, between the energy change dE and the change dA of the area of the horizon there is the following relationship [6]:

$$dE = \frac{\hbar \ln 2}{2\pi A_0 c} \kappa dA. \quad (10)$$

If one assumes that the horizon is a Rindler horizon one finds, using a similar chain of reasoning as Jacobson in his paper, that Eq.(10) implies Einstein's field equation for any radiation coming, from the accelerated observer's point of view, from the horizon. In other words, Eq.(10), when written for an arbitrary Rindler horizon, is just another expression for Einstein's field equation when the matter part consists purely of radiation. It can be written in the integral form:

$$\int_{E_i}^{E_f} \frac{dE}{\kappa(E)} = \frac{\hbar \ln 2}{2\pi A_0 c} (A_f - A_i), \quad (11)$$

where E_f and E_i , respectively, are the amounts of energy which may be extracted out from the horizon in the initial state i and the final state f , and A_i and A_f are the corresponding horizon areas. However, Postulate 1 states that the area of the horizon has a discrete spectrum with equal spacing. Because of that, we write our last postulate, a sort of "quantized Einstein equation", in the form:

Postulate 4: *If E_i and E_f are the initial and the final energies which may be extracted from the horizon, and $n_i A_0$ and $n_f A_0$ are the corresponding horizon areas, then*

$$\int_{E_i}^{E_f} \frac{dE}{\kappa(E)} = \frac{\hbar \ln 2}{2\pi c} (n_f - n_i), \quad (12)$$

where $\kappa(E)$ is the surface gravity of the horizon.

Among other things, this postulate quantizes the masses of black holes, and the energies of the quanta of emitted radiation.[7] Of course, the energies given by Eq.(12) must be red shifted, in curved spacetime and for accelerated observers, by the factor $(-g_{00})^{-1/2}$.

So far it may have remained somewhat obscure why we chose to identify, in Postulate 2, the microstates of the horizon as the different combinations of the areas of its parts. In the light of Postulates 3 and 4, however, everything becomes

clear: Postulate 4 implies that the energy of the emitted quantum of radiation depends on the accompanied change of the horizon area which, in turn, is always an integer times the fundamental area A_0 . Because the total area of the horizon is also a certain integer n times the fundamental area A_0 , it follows that the number of different combinations of the energies of the quanta of radiation is equal to the number –which we found to be 2^{n-1} – of the different combinations of the areas of the parts of the horizon. In other words, the radiation may come out from the horizon in 2^{n-1} ways. Because of that, the maximum amount of entropy carried by the radiation out from the horizon is $k_B \ln(2^{n-1})$. According to Postulate 3, however, the entropy of the horizon decreases exactly as much as the entropy of the radiation increases. Therefore the maximum entropy of the horizon, too, is $k_B \ln(2^{n-1})$. This may well provide the simplest conceivable explanation to the black hole entropy.

It should be noted that our postulates are assumed to be valid for any horizon, no matter whether that horizon is a black hole horizon, a de Sitter horizon, or a Rindler horizon. (If the horizon is infinite, our postulates are valid for any of its finite parts.) The postulates are in agreement with everything we know about the properties of horizons. Usually, the physical results concerning the horizons involved in our postulates are derived by means of arguments based on Einstein’s classical general relativity, and on the general principles of quantum mechanics and thermodynamics. The idea of this paper, however, has been to suggest that perhaps this line of reasoning should be turned upside down: Instead of trying to obtain the results from general relativity, we take these results to be the starting points, or postulates, of a microscopic theory of gravitation. Indeed, our extremely general and absurdly simple postulates concerning the microscopic properties of the horizons of spacetime imply, by means of Jacobson’s reasoning, Einstein’s general relativity in the classical limit, and they predict, among other things, the Hawking and the Unruh effects, together with the result that the entropy of a black hole is one quarter of its horizon area. No doubt, our postulates might have a certain taste of being rather *ad hoc*, nor do they say anything about the microscopic structure of spacetime. Because of that they should certainly not

be expected to be anywhere near the very fundamental, underlying postulates of quantum gravity. Nevertheless, one is perhaps not entirely able to avoid the feeling that, at the present state of research, our postulates satisfy many of the requirements one may reasonably pose for the postulates of a microscopic theory of gravity. It will be interesting to see whether a straightforward application of these postulates will predict new, so far unimagined, phenomena of nature.

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Notes and References

- [1] T. Jacobson, Phys. Rev. Lett. **75** 1260 (1995)
- [2] See, for example, A. Strominger and C. Vafa, Phys. Lett. **B379**, 99 (1996).
- [3] See, for example, A. Ashtekar, A. Baez, J. Korichi and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).
- [4] See, for example, R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (The University of Chicago Press, Chicago and London, 1994), and W. G. Unruh, Phys. Rev. **D14**, 870 (1976).
- [5] J. D Bekenstein, Lett. Nuovo Cimento **11**, 467 (1974). For an updated review and a list of references on this subject see, for example, J. D. Bekenstein, "The Case for Discrete Energy Levels of a Black Hole", hep-th/0107045. A comprehensive list of references may also be found in J. Mäkelä, P. Repo, M. Luomajoki and J. Piilonen, Phys. Rev. **D64**, 0240018 (2001).
- [6] To find the actually observed temperature of radiation and the energy change of the horizon, the quantity T of Eq.(9) as well as the quantity dE of Eq.(10) must be multiplied by an appropriate red shift factor. For an accelerated observer, for instance, this factor is, in natural units, the proper acceleration of the observer.

[7] Discrete mass eigenvalues and the thermal spectrum of black hole radiation do not necessarily contradict with each other. (See, for example, J. Mäkelä, Phys. Lett. **B390**, 115 (1997).)